NDIA

BOEING

# Statistical Modeling of Repairable Systems


Scott E. Black, Associate Technical Fellow

scott.e.black@boeing.com - 314-234-9037


***Tampa, Florida - Hyatt Regency Tampa***
***October 21 - 24, 2002***

NDIA

BOEING

# Overview

- Review of standard process models – assumptions/shortcomings
- Describe modulated process model
- Introduce statistical inference procedures
- Overview of simulation results
- Derivation for probability of mission success
- Summary

NDIA

BOEING

# Common Process Models

- Models for repairable systems must be able to describe the occurrence of events in time, and are thus inherently different from models non-repairable systems.

- Renewal Process: (*good – as – new*) A repaired unit is always brought to a like-new condition – time between failures are independent and identically distributed (iid). For this reason, the renewal process cannot be used to model a system experiencing deterioration or reliability improvement. (examples: Gamma).

- Non-homogeneous Poisson Process (NHPP): (*same – as – old* ) Following the repair, the system is returned to the state just prior to failure. (examples: Weibull / Power Law)

- In practice, neither process seems realistic. In many cases, a repaired unit is in better condition than it was just before failure, but still not in a like-new condition

NDIA

BOEING

# Modulated Process

- **Inhomogeneous gamma process (Berman 1981): Suppose that events, or shocks, occur according to an NHPP with intensity function u(t), and suppose that  a failure occurs not every shock but at every Kappa(th) shock, where Kappa is a positive integer.**

- **The joint probability density function for the first n failures is given by**

$$f(t_1, t_2, \ldots, t_n) = \left\{ \prod_{i-1}^{n} u(t_i)[U(t_i) - U(t_{i-1})]^{k-1} \right\} \text{ x } \frac{exp[-U(t_i)]}{[G(k)]^n}$$

  - **U(t) is the expected number of shocks before time t and is defined as**

$$U(t) = \int_0^t u(x)\, dx.$$

NDIA

BOEING

# Modulated Power Law Process

- If for example Kappa $(k)$ equaled 4, then every fourth shock would cause a failure

- A failed and repaired unit would be better than it was just before failure, since in order to cause another failure the required improvement parameter (Kappa) must accumulate to four again. A failed and repaired unit would not necessarily be as good as new.

- <u>Parameter definitions</u>
  - Kappa : measure if the improvement effected by the repair
  - Beta: is a measure of the system improvement or deterioration over the course of a systems life
  - Theta: Scaling parameter (units)

NDIA

BOEING

# Modulated Power Law Process (Special Cases)

- There are three special cases of the Modulated Power Law Process

Modulated Power Law Process

Gamma Renewal Process
*Like-New*
(Beta = 1)

Power Law Process
*Just prior to failure*
(kappa = 1)

Homogeneous Poisson Process
(Beta = Kappa = 1)

NDIA

BOEING

# Modulated Power Law Process
# Point Estimation

- If we take partial derivatives of the likelihood function with respect to theta,beta, and kappa we obtain the likelihood equations

$$l(\theta,\beta,\kappa) = -\left(\frac{t_n}{\theta}\right)^{\beta} + n\ln(\beta) - n\ln(\Gamma(\kappa)) - n\beta\kappa\ln\theta + (\beta-1)\sum_{i=1}^{n}\ln t_i + (\kappa-1)\sum_{i=1}^{n}\ln\left(t_i^{\beta} - t_{i-1}^{\beta}\right)$$

$$\frac{\partial l}{\partial \theta} = \left[\left(\frac{\mathrm{Beta}}{\mathrm{Theta}}\right)\cdot\left(\frac{t_n}{\mathrm{Theta}}\right)^{\mathrm{Beta}}\right] - \left[\frac{(n\cdot\mathrm{Beta}\cdot\mathrm{Kappa})}{\mathrm{Theta}}\right] = 0$$

$$\frac{\partial l}{\partial \beta} = -\left[\left(\frac{t_n}{\mathrm{Theta}}\right)^{\mathrm{Beta}}\cdot\left(\ln\left(\frac{t_n}{\mathrm{Theta}}\right)\right)\right] + \left(\frac{n}{\mathrm{Beta}}\right) - (n\cdot\mathrm{Kappa}\cdot\ln(\mathrm{Theta})) + \left(\sum_{j=1}^{n}\ln(t_j)\right) + \left[(\mathrm{Kappa}-1)\cdot\left[\sum_{k=1}^{n}\frac{\left[(t_k)^{\mathrm{Beta}}\cdot\ln(t_k)\right] - \left[(t_{k-1})^{\mathrm{Beta}}\cdot\ln(t_{k-1})\right]}{\left[(t_k)^{\mathrm{Beta}} - (t_{k-1})^{\mathrm{Beta}}\right]}\right]\right] = 0$$

$$\frac{\partial l}{\partial \kappa} = (-n\cdot\mathrm{Psi}(\mathrm{Kappa})) - (n\cdot\mathrm{Beta}\cdot\ln(\mathrm{Theta})) + \sum_{i=1}^{n}\ln\left[(t_i)^{\mathrm{Beta}} - (t_{i-1})^{\mathrm{Beta}}\right] = 0$$

- Here Psi denotes the di-gamma function $\psi(x) = \Gamma'(x) / \Gamma(x)$

NDIA

BOEING

# Modulated Power Law Process Asymptotic Confidence Intervals

- Without pivotal quantities, we must resort to asymptotic confidence intervals for the parameters. The asymptotic distribution of the estimator

$$\left[\hat{q}, \hat{b}, \hat{k}\right]$$

- Is multivariate normal with mean and covariance

$$m = \left[q, b, k\right] \qquad \Sigma = \left[J\left(q, b, k\right)\right]^{-1}$$

- Where the J matrix is the Jacobian and contains the second partial derivatives  of the likelihood function. Approximate confidence intervals for the parameters are given by

$$\hat{q} \pm z_{a/2}\sqrt{(1{,}1)entry\left[J\left(\hat{q}, \hat{b}, \hat{k}\right)\right]^{-1}} \qquad \hat{b} \pm z_{a/2}\sqrt{(2{,}2)entry\left[J\left(\hat{q}, \hat{b}, \hat{k}\right)\right]^{-1}} \qquad \hat{k} \pm z_{a/2}\sqrt{(3{,}3)entry\left[J\left(\hat{q}, \hat{b}, \hat{k}\right)\right]^{-1}}$$

NDIA

BOEING

# Modulated Power Law Process

## Simulation Results

### 95% Confidence Intervals (no transformation)

| Theta | Beta | Kappa | N | Theta CI % | Beta CI % | Kappa CI % |
|---|---|---|---|---|---|---|
| 200 | 0.75 | 1 | 10 | 77.6 | 89.7 | 97.1 |
| 200 | 0.75 | 1 | 20 | 85 | 93.9 | 95.8 |
| 200 | 0.75 | 1 | 30 | 86.8 | 92.9 | 95.5 |
| 200 | 1 | 1 | 10 | 81.2 | 92.4 | 97.6 |
| 200 | 1 | 1 | 20 | 87.1 | 93.8 | 96.6 |
| 200 | 1 | 1 | 30 | 89.6 | 92.9 | 95.8 |
| 200 | 1.5 | 1 | 10 | 80.2 | 91 | 96.7 |
| 200 | 1.5 | 1 | 20 | 90.1 | 93.7 | 96.2 |
| 200 | 1.5 | 1 | 30 | 91.5 | 96.6 | 95.4 |
| | | | | | | |
| Theta | Beta | Kappa | N | Theta CI % | Beta CI % | Kappa CI % |
| 200 | 0.75 | 2 | 10 | 74.8 | 89.8 | 98.1 |
| 200 | 0.75 | 2 | 20 | 85.3 | 92.7 | 96.3 |
| 200 | 0.75 | 2 | 30 | 85.1 | 93.1 | 96.4 |
| 200 | 1 | 2 | 10 | 78.5 | 88.4 | 98 |
| 200 | 1 | 2 | 20 | 88 | 93.3 | 96 |
| 200 | 1 | 2 | 30 | 90.7 | 94.1 | 96.3 |
| 200 | 1.5 | 2 | 10 | 81.8 | 88 | 98 |
| 200 | 1.5 | 2 | 20 | 88.7 | 93 | 95.7 |
| 200 | 1.5 | 2 | 30 | 90.1 | 92.2 | 96.9 |
| | | | | | | |
| | | | | | | |
| Theta | Beta | Kappa | N | Theta CI % | Beta CI % | Kappa CI % |
| 200 | 0.75 | 3 | 10 | 74.7 | 90.3 | 97.8 |
| 200 | 0.75 | 3 | 20 | 85 | 92.1 | 96.4 |
| 200 | 0.75 | 3 | 30 | 85.5 | 93.1 | 95.6 |
| 200 | 1 | 3 | 10 | 80.4 | 90.1 | 98.5 |
| 200 | 1 | 3 | 20 | 86.4 | 91.5 | 96.9 |
| 200 | 1 | 3 | 30 | 86.9 | 94.2 | 96 |
| 200 | 1.5 | 3 | 10 | 81.1 | 87.4 | 98 |
| 200 | 1.5 | 3 | 20 | 88.3 | 91.6 | 96.3 |
| 200 | 1.5 | 3 | 30 | 89.2 | 92.3 | 96.3 |

# NDIA

BOEING

# Modulated Power Law Process Asymptotic Confidence Intervals (log transformation - continued)

- **The approximate confidence intervals are therefore:**

$q:$

$$\left[ \hat{q}\exp\left\{ -z_{a/2}\sqrt{(1,1)\ entry\ in\ \left[J\left(\hat{q},\hat{b},\hat{k}\right)\right]^{-1}} \Big/ \hat{q} \right\},\ \hat{q}\exp\left\{ z_{a/2}\sqrt{(1,1)\ entry\ in\ \left[J\left(\hat{q},\hat{b},\hat{k}\right)\right]^{-1}} \Big/ \hat{q} \right\} \right]$$

$b:$

$$\left[ \hat{b}\exp\left\{ -z_{a/2}\sqrt{(2,2)\ entry\ in\ \left[J\left(\hat{q},\hat{b},\hat{k}\right)\right]^{-1}} \Big/ \hat{b} \right\},\ \hat{b}\exp\left\{ z_{a/2}\sqrt{(2,2)\ entry\ in\ \left[J\left(\hat{q},\hat{b},\hat{k}\right)\right]^{-1}} \Big/ \hat{b} \right\} \right]$$

$k:$

$$\left[ \hat{k}\exp\left\{ -z_{a/2}\sqrt{(3,3)\ entry\ in\ \left[J\left(\hat{q},\hat{b},\hat{k}\right)\right]^{-1}} \Big/ \hat{k} \right\},\ \hat{k}\exp\left\{ z_{a/2}\sqrt{(3,3)\ entry\ in\ \left[J\left(\hat{q},\hat{b},\hat{k}\right)\right]^{-1}} \Big/ \hat{k} \right\} \right]$$

NDIA

BOEING

# Modulated Power Law Process

## Simulation Results

### 95% Confidence Intervals (log transformation)

| Theta | Beta | Kappa | N | Theta CI % | Beta CI % | Kappa CI % |
|---|---|---|---|---|---|---|
| 200 | 0.75 | 1 | 10 | 86.3 | 85.5 | 85.8 |
| 200 | 0.75 | 1 | 20 | 89.9 | 89.6 | 89.7 |
| 200 | 0.75 | 1 | 30 | 91.5 | 92.5 | 92.9 |
| 200 | 1 | 1 | 10 | 87.6 | 88.1 | 86.2 |
| 200 | 1 | 1 | 20 | 90.9 | 90.7 | 92.6 |
| 200 | 1 | 1 | 30 | 90.5 | 91.5 | 91 |
| 200 | 1.5 | 1 | 10 | 85.7 | 87.1 | 85.8 |
| 200 | 1.5 | 1 | 20 | 91.9 | 91 | 91 |
| 200 | 1.5 | 1 | 30 | 91.9 | 91.8 | 92.4 |
| | | | | | | |
| Theta | Beta | Kappa | N | Theta CI % | Beta CI % | Kappa CI % |
| 200 | 0.75 | 2 | 10 | 88.7 | 87.9 | 85.8 |
| 200 | 0.75 | 2 | 20 | 93 | 92.3 | 91.8 |
| 200 | 0.75 | 2 | 30 | 92.5 | 93.2 | 91.4 |
| 200 | 1 | 2 | 10 | 88.5 | 87 | 87.2 |
| 200 | 1 | 2 | 20 | 93.3 | 92.4 | 93 |
| 200 | 1 | 2 | 30 | 93.1 | 93.2 | 93.9 |
| 200 | 1.5 | 2 | 10 | 88.6 | 85.7 | 84.2 |
| 200 | 1.5 | 2 | 20 | 91.8 | 90.9 | 91.5 |
| 200 | 1.5 | 2 | 30 | 92.7 | 92.6 | 91.9 |
| | | | | | | |
| | | | | | | |
| Theta | Beta | Kappa | N | Theta CI % | Beta CI % | Kappa CI % |
| 200 | 0.75 | 3 | 10 | 88.1 | 88.2 | 85 |
| 200 | 0.75 | 3 | 20 | 91.9 | 91 | 91.1 |
| 200 | 0.75 | 3 | 30 | 93.5 | 92.8 | 92.6 |
| 200 | 1 | 3 | 10 | 88.4 | 87.6 | 86.6 |
| 200 | 1 | 3 | 20 | 91.4 | 89.7 | 90.2 |
| 200 | 1 | 3 | 30 | 93.6 | 93.7 | 91.8 |
| 200 | 1.5 | 3 | 10 | 88 | 86.8 | 84.9 |
| 200 | 1.5 | 3 | 20 | 92.8 | 91.2 | 90.7 |
| 200 | 1.5 | 3 | 30 | 92.1 | 91.6 | 91.3 |

NDIA

BOEING

# Modulated Power Law Process

## Simulation Results

**Comparison of 95% Confidence Intervals (standard vs. log transformation) that include Kappa = 1**

| Theta | Beta | Kappa | N | Include Kappa =1 | Inlcude Kappa =1 (log) |
|---|---|---|---|---|---|
| 200 | 0.75 | 1 | 10 | 97.1 | 85.8 |
| 200 | 0.75 | 1 | 20 | 95.8 | 89.7 |
| 200 | 0.75 | 1 | 30 | 95.5 | 92.9 |
| 200 | 1 | 1 | 10 | 97.6 | 86.2 |
| 200 | 1 | 1 | 20 | 96.6 | 92.6 |
| 200 | 1 | 1 | 30 | 95.8 | 91 |
| 200 | 1.5 | 1 | 10 | 96.7 | 85.8 |
| 200 | 1.5 | 1 | 20 | 96.2 | 91 |
| 200 | 1.5 | 1 | 30 | 95.4 | 92.4 |
| | | | | | |
| 200 | 0.75 | 2 | 10 | 95.8 | 39.4 |
| 200 | 0.75 | 2 | 20 | 57.4 | 22.2 |
| 200 | 0.75 | 2 | 30 | 27.1 | 9.6 |
| 200 | 1 | 2 | 10 | 97.5 | 42 |
| 200 | 1 | 2 | 20 | 60.9 | 21 |
| 200 | 1 | 2 | 30 | 28.8 | 10 |
| 200 | 1.5 | 2 | 10 | 95.2 | 40.2 |
| 200 | 1.5 | 2 | 20 | 57.6 | 20.5 |
| 200 | 1.5 | 2 | 30 | 30 | 9.5 |
| | | | | | |
| 200 | 0.75 | 3 | 10 | 87.2 | 11.2 |
| 200 | 0.75 | 3 | 20 | 11 | 1.4 |
| 200 | 0.75 | 3 | 30 | 0.7 | 0.1 |
| 200 | 1 | 3 | 10 | 87.3 | 10.7 |
| 200 | 1 | 3 | 20 | 11.8 | 1 |
| 200 | 1 | 3 | 30 | 0.8 | 0 |
| 200 | 1.5 | 3 | 10 | 86.6 | 12.1 |
| 200 | 1.5 | 3 | 20 | 11.2 | 0.8 |
| 200 | 1.5 | 3 | 30 | 0.4 | 0.1 |

# Modulated Power Law Process Hypothesis Testing

- Previously we discussed the special cases of the MPLP. This leads to the following tests of hypothesis

$H_0 : k = 1, \; versus \; H_1 : k \neq 1$ (model reduces to the Power Law Process)

$H_0 : k = 1, \; versus \; H_1 : k \neq 1$ (model reduces to Gamma renewal process)

$H_0 : k = 1, \; versus \; H_1 : k \neq 1$ (model reduces to homogeneous Poisson Process)

- Since the exact distributions of the estimators are intractable, we rely on asymptotic results. The likelihood ratio test statistic is given by

$$LR = \frac{\max_{(q,b,k)\in S} L(q,b,k)}{\max_{(q,b,k)} L(q,b,k)}$$

# Modulated Power Law Process Hypothesis Testing (continued)

- **Test I:** $H_0 : k = 1$ **If the Null hypothesis is true, then the failure process is a power law process with parameters** $q, b$ **.**

$$LR_{PLP} = \frac{\max_{(q,b,k)\in S} L\left(\hat{q}_{PLP}, \hat{b}_{PLP}, 1\right)}{\max_{(q,b,k)} L\left(\hat{q}, \hat{b}, \hat{k}\right)}$$

- **Where** $\hat{b}_{PLP} = \dfrac{n}{\sum_{i=1}^{n-1} \log \dfrac{t_n}{t_i}}$ **and** $\hat{q} = \dfrac{t_n}{n^{\frac{1}{\hat{b}}}}$

- **Reject** $H_0 : k = 1$ **if** $-2 \log LR_{PLP} > c^2_{1-a}(1)$

NDIA

BOEING

# Modulated Power Law Process Hypothesis Testing (continued)

- **Test II:** $H_0 : k = 1$ **If the Null hypothesis is true, then the failure process is a gamma renewal (times between failures are iid random variables).** $GAM(k, q)$

$$LR_{GRP} = \frac{\max\limits_{(q,b,k)\in S} L\left(\hat{q}_{GRP}, 1, \hat{k}_{GRP}\right)}{\max\limits_{(q,b,k)} L\left(\hat{q}, \hat{b}, \hat{k}\right)}$$

- **The MLEs of theta and kappa do not have a closed form expression and must be solved by numerical methods. Differentiating the likelihood function and setting the results equal to zero leads to**

$$\hat{q} = \frac{\bar{x}}{\hat{k}} \qquad \log k - \Gamma'(k) / \Gamma(k) - \log\left(\bar{x} / \tilde{x}\right) \qquad \tilde{x} = \left(\prod x_i\right)^{1/n}$$

- **Reject** $H_0 : b = 1$ **if** $-2 \log LR_{GRP} > c^2_{1-a}(1)$

NDIA

BOEING

# Modulated Power Law Process Hypothesis Testing (continued)

- **Test III:** $H_0 : b = k = 1$ **If the Null hypothesis is true, then the failure process is a homogeneous Poisson process (times between failures are iid** $EXP(q)$ **random variables).**

$$LR_{HPP} = \frac{\max\limits_{(q,b,k)\in S} L\left(\hat{q}_{HPP},1,1\right)}{\max\limits_{(q,b,k)} L\left(\hat{q},\hat{b},\hat{k}\right)}$$

- **Where** $\hat{q}_{HPP} = \dfrac{t_n}{n}$

- **Reject** $H_0 : b = k = 1$ **if** $-2\log LR_{GRP} > c^2_{1-a}(2)$

NDIA

BOEING

# Modulated Power Law Process Simulation Results – Hypothesis Testing

**TEST:** $H_0 : b = 1 \quad H_a : b \neq 1$

| N | Theta | Beta | Kappa | Reject Ho |
|---|---|---|---|---|
| 20 | 200 | 1 | 3 | 6.00% |
| 20 | 200 | 1.25 | 3 | 45.70% |
| 20 | 200 | 1.5 | 3 | 88.60% |
| 20 | 200 | 2 | 3 | 99% |

| N | Theta | Beta | Kappa | Reject Ho |
|---|---|---|---|---|
| 30 | 200 | 1 | 3 | 5.90% |
| 30 | 200 | 1.25 | 3 | 67.30% |
| 30 | 200 | 1.5 | 3 | 97.40% |
| 30 | 200 | 2 | 3 | 98.4% |

Results of hypothesis test on Beta with alpha = .05

NDIA

BOEING

# Modulated Power Law Process Simulation Results – Hypothesis Testing

**TEST:** $H_0 : k = 1 \quad H_a : k \neq 1$

| N | Theta | Beta | Kappa | Reject Ho |
|---|---|---|---|---|
| 20 | 200 | 1.5 | 1 | 6.00% |
| 20 | 200 | 1.5 | 1.5 | 34.70% |
| 20 | 200 | 1.5 | 2 | 71.70% |
| 20 | 200 | 1.5 | 2.5 | 90.8% |
| 20 | 200 | 1.5 | 3 | 98.4% |
| 20 | 200 | 1.5 | 4 | 100% |

| N | Theta | Beta | Kappa | Reject Ho |
|---|---|---|---|---|
| 30 | 200 | 1.5 | 1 | 5.70% |
| 30 | 200 | 1.5 | 1.5 | 45.93% |
| 30 | 200 | 1.5 | 2 | 86.90% |
| 30 | 200 | 1.5 | 2.5 | 98.9% |
| 30 | 200 | 1.5 | 3 | 100.0% |
| 30 | 200 | 1.5 | 4 | 100.0% |

Results of hypothesis test on Kappa with alpha = .05

NDIA

BOEING

# Modulated Power Law Process Mission Readiness

- **From the definition of the Inhomogeneous Gamma Process**

$$f(t_1, t_2, \ldots . t_{n)} = \left\{ \prod_{i=1}^{n} l(t_i)[\Lambda(t_i) - \Lambda(t_{i-1})]^{K-1} \right\} \frac{\exp(-\Lambda(t_i))}{\Gamma(K)^n}$$

**where** $\Lambda(t) = \int_0^t l(t)dt$ **and** $l(t) = \left(\frac{b}{q}\right)\left(\frac{t}{q}\right)^{b-1}$

- **READINESS: Probability of no failures P( N=0) in a specified mission time given the current state of the system (conditional probability density function).**

$$f_n(t_n / t_{n-1}) = \lim_{\Delta t \to 0} \Pr\left(t < T_n < t + \Delta t / T_{n-1} = t\right)$$

NDIA

BOEING

# Modulated Power Law Process Mission Readiness (continued)

- **Using the intensity function for the PLP**

$$f_n(t/t_{n-1}) = \frac{\left\{\prod_{i=1}^{n}\left(\frac{b}{q}\right)\left(\frac{t_i}{q}\right)^{b-1}\left[\left(\frac{t_i}{q}\right)^b - \left(\frac{t_{i-1}}{q}\right)^b\right]^{k-1}\right\}\frac{\exp\left(-\left(t_n/q\right)^b\right)}{\Gamma(k)^n}}{\left\{\prod_{i=1}^{n-1}\left(\frac{b}{q}\right)\left(\frac{t_i}{q}\right)^{b-1}\left[\left(\frac{t_i}{q}\right)^b - \left(\frac{t_{i-1}}{q}\right)^b\right]^{k-1}\right\}\frac{\exp\left(-\left(t_{n-1}/q\right)^b\right)}{\Gamma(k)^{n-1}}}$$

- **Which reduces to**

$$f_n(t_n/t_{n-1}) = \frac{b t_n^{\,b-1}}{\Gamma(k) q^b}\left[\left(\frac{t_n}{q}\right)^b - \left(\frac{t_{n-1}}{q}\right)^b\right]^{k-1} \exp\left\{-\left[\left(\frac{t_n}{q}\right)^b - \left(\frac{t_{n-1}}{q}\right)^b\right]\right\}$$

NDIA

BOEING

# Modulated Power Law Process Mission Readiness (continued)

- **If we define**

$$a(t_n) = \left(\frac{t_n}{q}\right)^b - \left(\frac{t_{n-1}}{q}\right)^b$$

- **then**

$$f\left(t_n / t_{n-1}\right) = \frac{b}{q^b \Gamma(k)} t_n^{\,b-1} a(t_n)^{k-1} \exp\left(-a(t_n)\right)$$

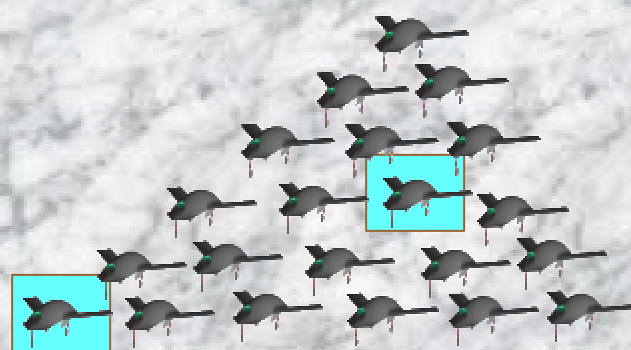
NDIA

BOEING

# Modulated Power Law Process Mission Readiness (continued)

- The probability of no failures in a given mission time is given by

$$\int_{MissionEndTime}^{\infty} \frac{b}{q^{b}\,\Gamma(k)}\, t_n^{\,b-1} a(t_n)^{k-1} \exp\left(-a(t_n)\right) dt_n$$



- **Select required aircraft with highest probability of mission completion**

NDIA

BOEING

# Summary

- Modulated Power Law Process – provides ability to model improvement into repairable systems.
- Inference procedures capable of detecting special cases.
- Asymptotic confidence intervals were very effective in simulation study ( nominal level ) for sample size > 30.
- Conditional distribution presented - Estimating probability of failure in a given mission time (readiness).
- Modulated Power Law Process provides insight into the overall support process. Kappa can be used as support improvement measure.